Class 12 : Mathematics

Section A

1) Given $\vec{a}\cdot\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$|\vec{b}| = 7$ Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \dfrac{8}{7}$

2) Here $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$ $\quad |\vec{a}| = 7$

Unit vector $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{2\hat{i} - 6\hat{j} + 3\hat{k}}{7}$

$= \frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$

3) $I = \displaystyle\int \frac{\sec^2 x}{3+\tan x}\,dx$ put $3 + \tan x = t$, $\sec^2 x\,dx = dt$

$I = \displaystyle\int \frac{dt}{t} = \log|t| + c = \log|3+\tan x| + c$

4. We know $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\frac{\pi}{2} + \cos^2\frac{\pi}{3} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{1}{4}$

$\cos^2\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi/6$

Section B

5. $y = f(e^{\sin^{-1}2x})$ $\quad \dfrac{dy}{dx} = f'(e^{\sin^{-1}2x})\cdot e^{\sin^{-1}2x}\cdot\dfrac{1}{\sqrt{1-4x^2}}\times 2.$

$= \dfrac{2e^{\sin^{-1}2x}}{\sqrt{1-4x^2}}\, f'(e^{\sin^{-1}2x}).$

6. Let x denote the edge. v denote Volume.

$\dfrac{dv}{dt} = 9\,cm^3/sec$, $x = 10$ cm

$V = x^3$, $\dfrac{dv}{dt} = 3x^2\cdot\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = \dfrac{9}{3x^2}$

$S = 6x^2$, $\dfrac{ds}{dt} = 12x\cdot\dfrac{dx}{dt} = 12\times 10\times\dfrac{9}{3\times 10\times 10} = \dfrac{36}{10} = 3.6\,cm^2/sec.$

$\int \frac{}{1+\sin x} \quad \int \frac{}{1-\sin^2 x} \quad \int \frac{}{\cos^2 x} \quad \int \frac{}{\cos^2 x}\,dx$

$= \int \sec^2 x\,dx - \int \sec x \tan x\,dx = \tan x - \sec x + C.$

8. $y = a\cos 2x + b\sin 2x.$

$\frac{dy}{dx} = -a\sin 2x \times 2 + b\cos 2x \times 2 = 2[-a\sin 2x + b\cos 2x]$

$\frac{d^2y}{dx^2} = 2[-a\cos 2x \cdot 2 + b(-\sin 2x)\cdot 2] = -4[a\cos 2x + b\sin 2x]$

$\frac{d^2y}{dx^2} = -4y \Rightarrow \frac{d^2y}{dx^2} + 4y = 0$

9. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 1 & -1 \end{vmatrix} = i(-2+1) - j(-1+3) + k(1-6)$

$= -i - 2j - 5k.$

$|\vec{a} \times \vec{b}| = \sqrt{30}$

10. Let $A(2,3,4)$, $B(-1,-2,1)$, $C(5,8,7)$

D.r of $AB = (-3\ -5\ -3)$ Dr's of AB and AC

Dr of $AC = (3\ 5\ 3)$ are proportional

But A is common $\therefore$ A, B, C are collinear.

11) Let the number of pieces of model A to be manufactured be x. and of B be y.

Then L.P.P is maximise $P = 8000x + 12000y$

Subject to $9x + 12y \le 180 \Rightarrow 3x + 4y \le 60$

$x + 3y \le 30$; $x, y \ge 0$

$P(E\cup F) = P(E)+P(F) - P(E)\times P(F)$

$0.85 = P(E) + 0.35 - P(E)\times 0.35$

$0.85 = P(E)(1-0.35) + 0.35 = P(E)\cdot 0.65 + 0.35$

$P(E) = \frac{0.52}{0.65} = \frac{10}{13}$ $\therefore P(F) = 1 - \frac{10}{13} = \frac{3}{13}.$

Section C.

13. $f(x)$ is continuous at $x=0$

$L.H.L = f(0) = R.H.L.$

$L.H.L = f(0) \Rightarrow \lim_{x\to 0} f(x) = \frac{2(0)+1}{0-1}$

$\Rightarrow \lim_{x\to 0} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} = -1 \Rightarrow \lim_{x\to 0} \frac{(\sqrt{1+kx})^2-(\sqrt{1-kx})^2}{x(\sqrt{1+kx}+\sqrt{1-kx})} = -1$

$\Rightarrow \lim_{x\to 0}\left(\frac{2kx}{x\sqrt{1+kx}+\sqrt{1-kx}}\right) = -1 \Rightarrow \lim_{x\to 0}\left(\frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}}\right) = -1$

$\Rightarrow \frac{2k}{2} = -1 \Rightarrow k = -1$ $f(x)$ is continuous at $x=0$ if $k=-1$

14. Let $u = x^{\sin x} \Rightarrow \log u = \sin x \log x \Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = \sin x\cdot\frac{1}{x} + \log x(\cos x)$

$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x\log x\right]$

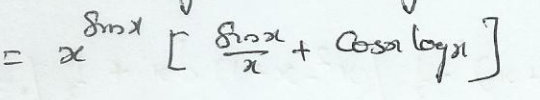
$v = \sin x^{\cos x} \Rightarrow \log v = \cos x \log \sin x \Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \cos x\cdot\frac{1}{\sin x}\cos x + \log\sin x(-\sin x)$

$\frac{dv}{dx} = \sin x^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x\log\sin x\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x\log x\right] + \sin x^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x\log x\right]$

15. $f(x) = 6x^3 - 12x^2 - 90x = 6x(x-5)(x+3)$

$f'(x) = 0 \Rightarrow x = 5$ or $x = -3$, $x = 0$



$f'(x) > 0$, $\forall x \in (-3,0)\cup(5,\infty) \Rightarrow$ strictly increasing

$f'(x) < 0$ $\forall x \in (-\infty,-3)\cup(0,5) \Rightarrow$ strictly decreasing

Consider $\frac{x^2}{x^4+x^2-2}$ and put $x^2=y$.

$$\frac{x^2}{x^4+x^2-2} = \frac{y}{y^2+y-2} = \frac{y}{(y+2)(y-1)}$$

$$\frac{y}{(y+2)(y-1)} = \frac{A}{y+2}+\frac{B}{y-1} \Rightarrow A=\frac{2}{3} \quad B=\frac{1}{3}$$

$$\int \frac{x^2}{(x^2+2)(x^2-1)}\,dx = \frac{2}{3}\int\frac{1}{x^2+2}\,dx + \frac{1}{3}\int\frac{1}{x^2-1}\,dx$$

$$= \frac{2}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right] + \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|\right]+c$$

$$= \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{1}{6}\log\left|\frac{x-1}{x+1}\right|+c.$$

16.

OR

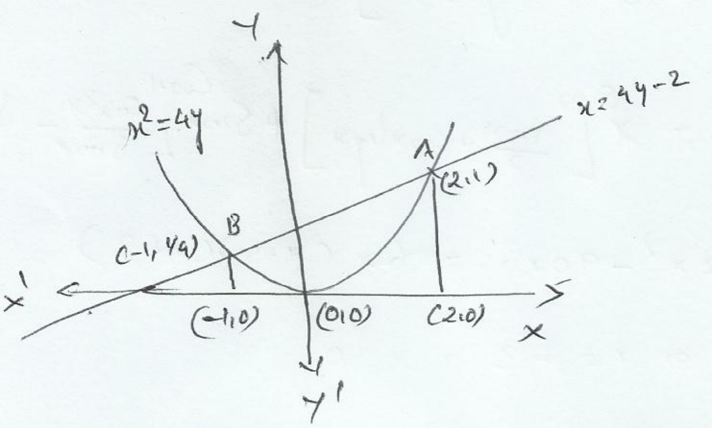
$$I=\int x^2\log(1+x)\,dx = \log(1+x)\cdot\frac{x^3}{3} - \int\frac{x^3}{3(1+x)}\,dx.$$

$$= \frac{x^3}{3}\log(1+x) - \frac{1}{3}\int\frac{x^3+1-1}{1+x}\,dx = \frac{x^3}{3}\log(1+x) - \frac{1}{3}\int\frac{x^3+1}{x+1}\,dx + \frac{1}{3}\int\frac{1}{x+1}\,dx.$$

$$= \frac{x^3}{3}\log(1+x) - \frac{1}{3}\int(x^2-x+1)\,dx + \frac{1}{3}\int\frac{1}{x+1}\,dx.$$

$$= \frac{x^3}{3}\log(1+x) - \frac{x^3}{9} + \frac{x^2}{6} + \frac{x}{3} + \frac{1}{3}\log|1+x| + c.$$

17.



Required Area

$$= \int_{-1}^{2}\frac{x+2}{4}\,dx - \int_{-1}^{2}\frac{x^2}{4}\,dx$$

$$= \frac{1}{4}\left[\frac{(x+2)^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}.$$

$$= \frac{1}{4}\left[\frac{16}{3}-\frac{5}{6}\right] = \frac{27}{24} = \frac{9}{8} \text{ sq units}$$

$x\frac{dy}{dx} + y = x\cos x + \sin x.$

$\frac{dy}{dx} + \frac{1}{x}\cdot y = \cos x + \frac{1}{x}\sin x.$ $\quad I.F = e^{\int \frac{1}{x} dx} = e^{\log x} = x.$

Slouhon $xy = \int (x\cos x + \sin x)\, dx \Rightarrow xy = x\sin x + C.$

$y = \sin x + C.\frac{1}{x}.$ When $x = \frac{\pi}{2},\ y = 1 \Rightarrow 1 = 1 + C.\frac{2}{\pi}$

$\Rightarrow C = 0$

$\therefore$ The soln is $y = \sin x.$

19. $\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k},\quad \vec{a} - \vec{b} = -4\hat{i} + (7-\lambda)\hat{k}$

Since $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are $\perp^{er}$ $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 0$

$(6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}) \cdot (-4\hat{i} + (7-\lambda)\hat{k}) = 0$

$\Rightarrow -24 + (7+\lambda)(7-\lambda) = 0 \Rightarrow 49 - \lambda^2 - 24 = 0$

$\Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$

OR

If A, B, C, D are coplanar

$\vec{AB} \cdot (\vec{BC} \times \vec{CD}) = 0$ (triple product is 0)

$\vec{AB} = \hat{i} + (x-1)\hat{j} + 4\hat{k},\quad \vec{BC} = 0\hat{i} + (1-x)\hat{j} - 7\hat{k}$

$\vec{CD} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{AB} \cdot (\vec{BC} \times \vec{CD}) = \begin{vmatrix} 1 & x-1 & 4 \\ 0 & 1-x & -7 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$1(1 - x + 21) - (x-1)14 + 4[2(x-1)] = 0$

$22 - x - 14x + 14 + 8x - 8 = 0$

$-7x = -28 \Rightarrow x = 4$

Since $\vec{d}$ is $\perp^{er}$ to both $\vec{a}$ & $\vec{b}$

$\vec{d}\cdot\vec{a} = 0$ and $\vec{d}\cdot\vec{b} = 0$

$x + 4y + 2z = 0$ —① Given $\vec{c}\cdot\vec{d} = 15$

$3x - 2y + 7z = 0$ —② $\Rightarrow 2x - y + 4z = 15$ —③

Solving ①, ② and ③ $x = \frac{160}{3}$, $y = -\frac{5}{3}$, $z = -\frac{70}{3}$

$\vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} \Rightarrow \vec{d} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$

21. $L_1 : \vec{r} = (i + 2j + k) + \lambda(i - j + k)$

$L_2 : \vec{r} = (2i - j - k) + \mu(2i + j + 2k)$

$\vec{a_1} = i + 2j + k$, $b_1 = i - j + k$ $\quad \vec{a_2} - \vec{a_1} = \hat{i} - 3\hat{j} - 2k$

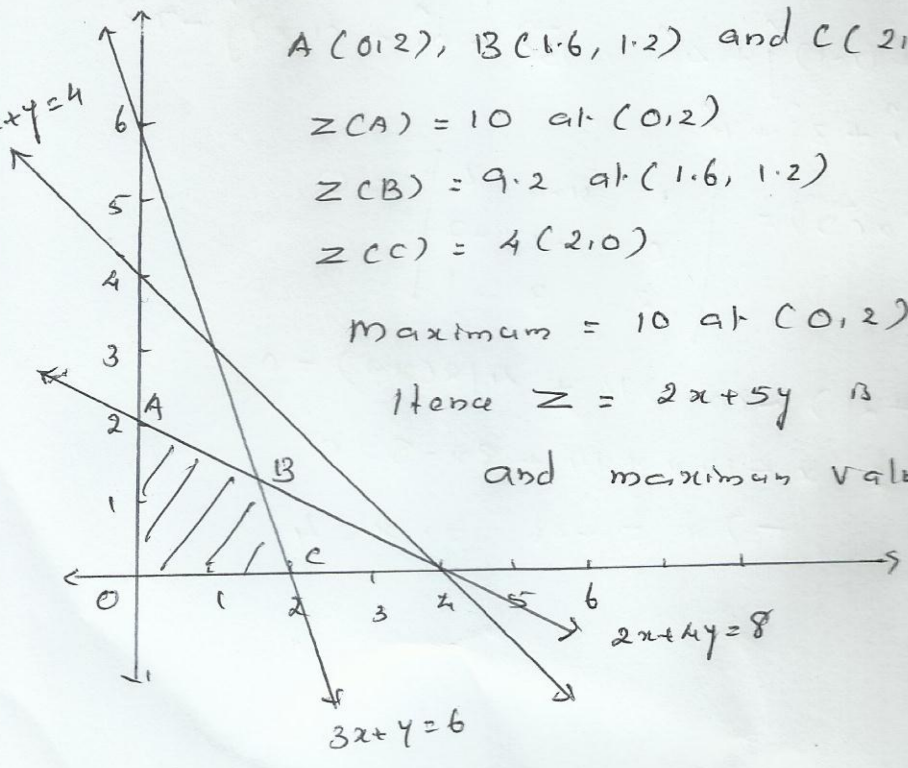
$\vec{a_2} = 2i - j - k$, $b_2 = 2i + j + 2k$

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 3\hat{k} \quad |\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}.$

$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -3 - 6 = -9$

Shortest distance $= \dfrac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|(\vec{b_1} \times \vec{b_2})|} = \left|\dfrac{-9}{3\sqrt{2}}\right| = \dfrac{3\sqrt{2}}{2}$

22.



A (0, 2), B (1.6, 1.2) and C (2, 0)

Z(A) = 10 at (0, 2)

Z(B) = 9.2 at (1.6, 1.2)

Z(C) = 4 (2, 0)

Maximum = 10 at (0, 2)

Hence $Z = 2x + 5y$ is max at A (0, 2)

and maximum value is 10

23. Let E and F denote the events P speaks the truth, Q speaks the truth

$P(E) = \frac{70}{100}$ $\quad P(\bar{E}) = 1 - P(E) = 1 - \frac{70}{100} = \frac{30}{100}$

$P(F) = \frac{80}{100}$ $\quad P(\bar{F}) = 1 - P(F) = 1 - \frac{80}{100} = \frac{20}{100}$

Required probability $= P(E)P(F) + P(\bar{E})P(\bar{F})$

$= \frac{70}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{20}{100}$

$= \frac{56}{100} + \frac{6}{100} = \frac{62}{100} = 62\%$

OR

Let x is the random variable denoting the number of kings. x can take 0, 1 or 2.

$P(x=0) = P(\text{no king}) \times P(\text{no king}) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$

$P(x=1) = P(\text{king and no king}) + P(\text{no king and king})$

$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} = \frac{24}{169}$

$P(x=2) = P(\text{king}) \times P(\text{king}) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

x	0	1	2
$p(x)$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

mean $= \sum_{i=0}^{2} x_i p(x_i)$

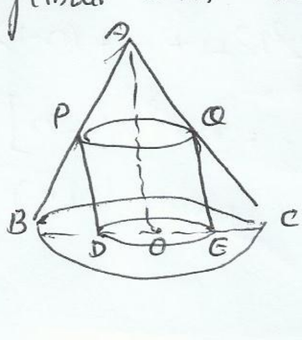
$= 0 \times \frac{144}{169} + 1 \times \frac{24}{169} + 2 \times \frac{1}{169}$

$= \frac{26}{169} = \frac{2}{13}$.

Section D

24. Let $OC = r$ be the radius of the cone and $OA = h$ (height)

Let a cylinder with radius $OE = x$ inscribed in the given cone.



$\frac{QE}{OA} = \frac{EC}{OC}$

$\frac{QE}{h} = \frac{r-x}{r} \Rightarrow QE = \frac{h(r-x)}{r}$

then $S(x) = \frac{2\pi x h(r-x)}{r} = \frac{2\pi h}{r}(rx - x^2)$

$S'(x) = \frac{2\pi h}{r}(r-2x)$ $S''(x) = -\frac{4\pi h}{r}$

Now $S'(x) = 0$ gives $x = \frac{r}{2}$ Since $S''(x) < 0 \ \forall x$.

$x = \frac{r}{2}$ is a point of maxima of S.

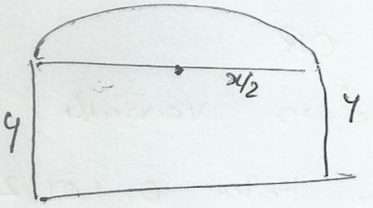
OR

Let side of box $= x$ cm. height $= h$ cm. $V = 1024$

$x \cdot x \cdot h = 1024 \Rightarrow h = \frac{1024}{x^2}$, $C = 5(2x^2) + 2.5(4xh)$

$= 10x^2 + 10hx$.

$= 10x^2 + \frac{10240}{x}$



$\frac{dC}{dx} = 20x - \frac{10240}{x^2}$

$\frac{dc}{dx} = 0 \Rightarrow x^3 = 512 \Rightarrow x = 8$

$\frac{d^2c}{dx^2} = 20 + \frac{2(10240)}{x^3}$ $\left.\frac{d^2c}{dx^2}\right|_{x=8} > 0$

Cost is least at $x = 8$ and least Cost $= 10 \times 8^2 + \frac{10240}{8}$

$= ₹\ 1920$

25. $I = \int_0^2 (x^2+1)\,dx$.

$\int_a^b (x^2+1)\,dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + \cdots + f(a+\overline{n-1}h)\right]$

$a = 0, b = 2$ $f(x) = x^2+1$, $nh = b - a = 2 - 0 = 2$

$f(a) = f(0) = 1$, $f(a+h) = f(0+h) = f(h) = h^2+1$

$f(a+2h) = f(0+2h) = f(2h) = 2^2h^2+1$

— — — — — — —

$f(a+(n-1)h) = f(0+(n-1)h) = f[(n-1)h] = (n-1)^2h^2+1$

$I = \lim_{h \to 0} h\left[1 + h^2+1 + 2^2h^2+1 + \cdots + (n-1)^2h^2+1\right]$

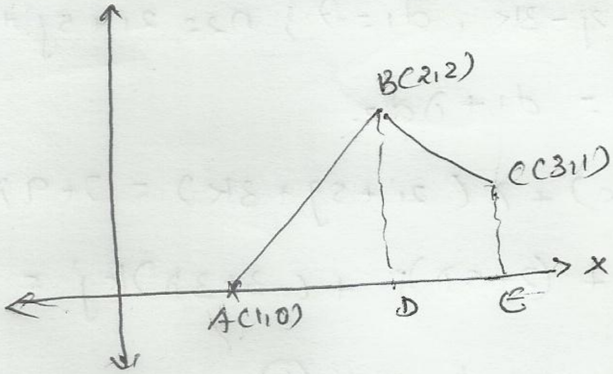
$= \lim_{h \to 0} h\left[h^2 + 2^2h^2 + \cdots + (n-1)^2h^2 + n\right]$

$= \lim_{h \to 0} h\left(h^2\left[1^2 + 2^2 + \cdots + (n-1)^2\right] + n\right)$

$$= \lim_{h\to 0}\left[\frac{(nh-h)\, nh\,(2nh-h)}{6}+nh\right] = \lim_{h\to 0}\left[\frac{(2-h)\,2\,(4-h)}{6}+2\right]$$

$$= \frac{14}{3}.$$

26.



Let A(1,0), B(2,2), C(3,1) be the vertices of a triangle ABC.

Area of $\triangle ABC$ = Area of $\triangle ABD$ + area of trap BDEC − area of $\triangle AEC$.

Now eqn of sides AB, BC and CA are given by

$y = 2(x-1)$, $y = 4-x$, $y = \frac{1}{2}(x-1)$

$$\text{area of } \triangle ABC = \int_1^2 2(x-1)\,dx + \int_2^3 (4-x)\,dx - \int_1^3 \frac{x-1}{2}\,dx$$

$$= 2\left[\frac{x^2}{2}-x\right]_1^2 + \left[4x-\frac{x^2}{2}\right]_2^3 - \frac{1}{2}\left[\frac{x^2}{2}-x\right]_1^3$$

$$= \frac{3}{2}.$$

27. $(3xy+y^2)\,dx + (x^2+xy)\,dy = 0$

$$\frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy} \qquad \text{put } y = vx,\ \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$$v + x\frac{dv}{dx} = \frac{3x\,vx + v^2x^2}{x^2+x\cdot vx} = -\frac{(3v+v^2)}{1+v}$$

$$x\cdot\frac{dv}{dx} = -\frac{(3v+v^2)}{1+v} - v = \frac{-4v-2v^2}{1+v} = \frac{-(2v^2+4v)}{1+v}$$

$$\frac{1+v}{2v^2+4v}\,dv = -\frac{1}{x}\,dx.$$

Integrating $\frac{1}{4}\log|2v^2+4v| + \log|x| = \log c_1$

$$(2v^2+4v)^{1/4}x = c_1$$

$\Rightarrow 2x^2y^2 + 4x^3y = c$. when $x=1, y=1$ $c=6$

$\therefore 2x^2y^2 + 4x^3y = 6 \Rightarrow x^2y^2 + 2x^3y = 3$.

28. Here $\vec{n_1} = 2i + 2j - 3k$, $d_1 = 7$; $n_2 = 2i + 5j + 3k$, $d_2 = 9$

$\vec{r} \cdot (\vec{n_1} + \lambda \vec{n_2}) = d_1 + \lambda d_2$

$\vec{r} \cdot (2i + 2j - 3k) + \lambda(2i + 5j + 3k) = 7 + 9\lambda$

$\vec{r} \cdot (2+2\lambda)i + (2+5\lambda)j + (-3+3\lambda)kj = 7 + 9\lambda$ —— ①

$(2,1,3)$ is a point on ①

$(2i + j + 3k) \cdot (2+2\lambda)i + (2+5\lambda)j + (-3+3\lambda)k = 7 + 9\lambda$

$2(2+2\lambda) + (2+5\lambda) + 3(-3+3\lambda) = 7 + 9\lambda$.

$9\lambda - 3 = 7 \Rightarrow \lambda = \frac{10}{9}$

Substituting λ in ①

$\vec{r} \cdot \left[2 + 2\left(\frac{10}{9}\right) i + \left(2 + 5 \times \frac{10}{9}\right) j + \left(-3 + 3\left(\frac{10}{9}\right)\right) k \right] = 7 + 9\left(\frac{10}{9}\right)$

$\vec{r} \cdot \left(\frac{38}{9} i + \frac{68}{9} j + \frac{3}{9} \hat{k} \right) = 17$

$\vec{r} \cdot (38i + 68j + 3\hat{k}) = 153$ is the required eqn.

OR

Eqn of plane passing through three non-collinear points.

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-3 & y+1 & z-2 \\ 5-3 & 2+1 & 4-2 \\ -1-3 & -1+1 & 6-2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$(x-3)(12-0) - (y+1)16 + (z-2)(0+12) = 0$

$3x - 4y + 3z - 19 = 0$ —— ①

$\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) - 19 = 0$

We know distance of $P(6,5,9)$ from the plane (1) is given by $\frac{|3\times 6 - 4\times 5 + 3\times 9 - 19|}{\sqrt{3^2 + (-4)^2 + 3^2}} = \frac{6}{\sqrt{34}}$ units.

29. Let E_1: lost card is a diamond

E_2: lost card is not a diamond

A: getting 2 diamond cards from 51 cards

E_1 and E_2 are mutually exclusive.

$P(E_1) = \frac{13}{52} = \frac{1}{4}$ $P(E_2) = \frac{39}{52} = \frac{3}{4}$

$P(A/E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12\times 11}{51\times 50}$, $P(A/E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13\times 12}{51\times 50}$

By Baye's theorem.

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{4}\left(\frac{12\times 11}{51\times 50}\right)}{\frac{1}{4}\left(\frac{12\times 11}{51\times 50}\right) + \frac{3}{4}\left(\frac{13\times 12}{51\times 50}\right)}$$

$$= \frac{12\times 11}{12\times 11 + 13\times 12\times 3} = \frac{11}{11 + 13\times 3} = \frac{11}{50}$$

OR.

$n = 50$, $p = \frac{1}{100}$ $q = 1 - p = \frac{99}{100}$

Let X be the no of wins.

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$$

$$= 1 - {}^{50}C_0\, p^0 q^{50}$$

$$= 1 - \left(\frac{99}{100}\right)^{50}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49} = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\left(\frac{99}{100}\right)^{50} + \frac{50}{100}\left(\frac{99}{100}\right)^{49}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{50}{100}\right]$$

$$= 1 - \frac{149}{100}\left(\frac{99}{100}\right)^{49}$$